OCR RECOGNISING ACHIEVEMENT

# Tuesday 17 January 2012 – Morning

## A2 GCE MATHEMATICS

**4724** Core Mathematics 4

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**
- Printed Answer Book 4724
- List of Formulae (MF1)

**Other materials required:**
- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

*4732820112*

## INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

## INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

## INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

**1** When the polynomial f($x$) is divided by $x^2 + 1$, the quotient is $x^2 + 4x + 2$ and the remainder is $x - 1$. Find f($x$), simplifying your answer. **[3]**

**2** **(i)** Find, in the form $\mathbf{r} = \mathbf{a} + t\mathbf{b}$, an equation of the line $l$ through the points (4, 2, 7) and (5, –4, –1). **[3]**

**(ii)** Find the acute angle between the line $l$ and a line in the direction of the vector $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$. **[4]**

**3** The equation of a curve $C$ is $(x + 3)(y + 4) = x^2 + y^2$.

**(i)** Find $\frac{dy}{dx}$ in terms of $x$ and $y$. **[4]**

**(ii)** The line $2y = x + 3$ meets $C$ at two points. What can be said about the tangents to $C$ at these points? Justify your answer. **[2]**

**(iii)** Find the equation of the tangent at the point (6, 0), giving your answer in the form $ax + by = c$, where $a$, $b$ and $c$ are integers. **[2]**

**4** **(i)** Expand $(1 - 4x)^{\frac{1}{4}}$ in ascending powers of $x$, up to and including the term in $x^3$. **[5]**

**(ii)** The term of lowest degree in the expansion of

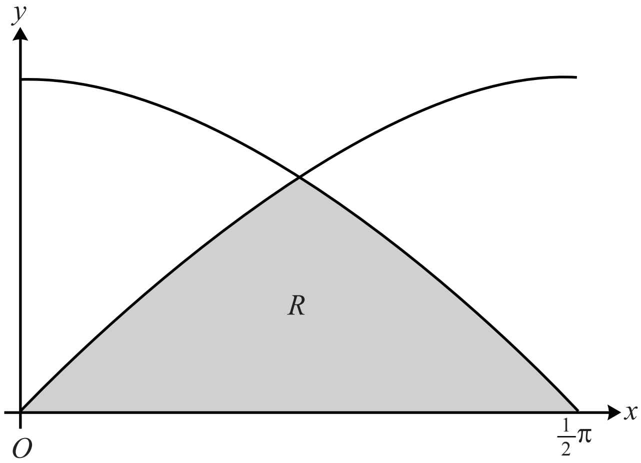
$$(1 + ax)(1 + bx^2)^7 - (1 - 4x)^{\frac{1}{4}}$$

in ascending powers of $x$ is the term in $x^3$. Find the values of the constants $a$ and $b$. **[4]**

**5** Use the substitution $u = \cos x$ to find the exact value of

$$\int_0^{\frac{1}{3}\pi} \sin^3 x \cos^2 x \, dx .$$ **[6]**

**6**




The diagram shows the curves $y = \cos x$ and $y = \sin x$, for $0 \leqslant x \leqslant \frac{1}{2}\pi$. The region $R$ is bounded by the curves and the $x$-axis. Find the volume of the solid of revolution formed when $R$ is rotated completely about the $x$-axis, giving your answer in terms of $\pi$. **[7]**

**7** The equation of a straight line $l$ is

$$\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} + t\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}.$$

$O$ is the origin.

**(i)** Find the position vector of the point $P$ on $l$ such that $OP$ is perpendicular to $l$. **[3]**

**(ii)** A point $Q$ on $l$ is such that the length of $OQ$ is 3 units. Find the two possible position vectors of $Q$. **[3]**

**8** A curve is defined by the parametric equations

$$x = \sin^2\theta, \quad y = 4\sin\theta - \sin^3\theta,$$

where $-\frac{1}{2}\pi \leqslant \theta \leqslant \frac{1}{2}\pi$.

**(i)** Show that $\dfrac{dy}{dx} = \dfrac{4 - 3\sin^2\theta}{2\sin\theta}$. **[3]**

**(ii)** Find the coordinates of the point on the curve at which the gradient is 2. **[3]**

**(iii)** Show that the curve has no stationary points. **[2]**

**(iv)** Find a cartesian equation of the curve, giving your answer in the form $y^2 = f(x)$. **[2]**

**[Questions 9 and 10 are printed overleaf.]**

**9** Find the exact value of $\int_0^1 (x^2 + 1)e^{2x}\,dx$. **[7]**

**10** **(i)** Write down the derivative of $\sqrt{y^2 + 1}$ with respect to $y$. **[1]**

**(ii)** Given that $\frac{dy}{dx} = \frac{(x-1)\sqrt{y^2+1}}{xy}$ and that $y = \sqrt{e^2 - 2e}$ when $x = e$,

find a relationship between $x$ and $y$. **[8]**

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| **1** | | | f($x$) = $\left(x^2+1\right)\left(x^2+4x+2\right)+\left(x-1\right)$ | M1 | written or clearly intended | (Alt)Long div with 3 stages/equate quots/equate rems |
| | | | $x^4+4x^3+...$ | B1 | | |
| | | | + … $3x^2+5x+1$ | A1 | | |
| | | | | **[3]** | | |
| **2** | (i) | | **a** = $\begin{pmatrix}4\\2\\7\end{pmatrix}$ or $\begin{pmatrix}5\\-4\\-1\end{pmatrix}$ | B1 | Accept any notation | |
| | | | **b** = Difference between the two points | M1 | | |
| | | | Provided final answer is of form <u>**r** = **a** + *t***b**</u> | A1 | | |
| | | | $\begin{pmatrix}1\\-6\\-8\end{pmatrix}$ or $\begin{pmatrix}-1\\6\\8\end{pmatrix}$ | | | |
| | | | | **[3]** | | |
| **2** | (ii) | | Method for magnitude of <u>any</u> vector | M1 | Accept e.g. $\sqrt{1^2-6^2-8^2}$ | |
| | | | Method for scalar product of <u>any</u> 2 vectors | M1 | | |
| | | | Using $\cos\theta=\dfrac{\mathbf{c.d}}{\lvert\mathbf{c}\rvert\lvert\mathbf{d}\rvert}$ for their **b** and $\begin{pmatrix}1\\2\\3\end{pmatrix}$ | M1 | | |
| | | | 21.4 or better (21.444513); 0.374 or better (0.374277) | A1 | | |
| | | | | **[4]** | | |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| **3** | (i) | | Treat $(x+3)(y+4)$ or $xy$ as a product | M1 | attempting $u.\mathrm{d}v+v.\mathrm{d}u$ | |
| | | | $\frac{\mathrm{d}}{\mathrm{d}x}(x+3)(y+4)=(x+3)\frac{\mathrm{d}y}{\mathrm{d}x}+(y+4)$ or $\frac{\mathrm{d}}{\mathrm{d}x}(xy)=x\frac{\mathrm{d}y}{\mathrm{d}x}+y$ | A1 | | |
| | | | $\frac{\mathrm{d}}{\mathrm{d}x}(y^2)=2y\frac{\mathrm{d}y}{\mathrm{d}x}$ | B1 | | |
| | | | $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{2x-y-4}{x-2y+3}$ | B1 **[4]** | AEF including $-\frac{a}{b},\frac{-a}{b},\frac{a}{-b}$ | |
| **3** | (ii) | | State or imply that denominator is zero | B1 | Provided denom is $x-2y+3$ or $-x+2y-3$ | |
| | | | Tangents are parallel to $y$-axis | B1 **[2]** | Accept vertical or of the form $x = k$ | |
| **3** | (iii) | | Substitute (6,0) into their $\frac{\mathrm{d}y}{\mathrm{d}x}$ $(=\frac{8}{9})$ | M1 | | |
| | | | $8x-9y=48$ FT $fx-gy=6f$ | A1 FT **[2]** | FT their numerical $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{f}{g}$ www in this part | |
| **4** | (i) | | First two terms in expansion = $1-x$ | B1 | (simplify to this, now or later) | |
| | | | Third term shown as $\frac{\frac{1}{4}.-\frac{3}{4}}{2}(-4x)^2$ | M1 | $-\frac{3}{4}$ can be $\frac{1}{4}-1$; $(-4x)^2$ can be $-4x^2$ or $-16x^2$ | |
| | | | $=-\frac{3}{2}x^2$ | A1 | | |
| | | | Fourth term shown as $\frac{\frac{1}{4}.-\frac{3}{4}.-\frac{7}{4}}{2.3}(-4x)^3$ | M1 | Similar allowances as for first M1 | |
| | | | $=-\frac{7}{2}x^3$ | A1 **[5]** | [Complete expansion is $1-x-\frac{3}{2}x^2-\frac{7}{2}x^3...$] | |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| 4 | (ii) | | $\left(1+bx^2\right)^7$ shown (implied) as $1+7bx^2+...$ | B1 | | |
| | | | Clear indic that terms involving $x$ and $x^2$ must cancel | M1 | | |
| | | | $a=-1$ | A1 FT | If **(i)** = $1+\lambda x+\mu x^2$, $a=\lambda$ | |
| | | | $b=-\frac{3}{14}$ | A1 FT | If **(i)** = $1+\lambda x+\mu x^2$, $b=\frac{1}{7}\mu$ | |
| | | | | | FT from wrong **(i)** only, not wrong $\left(1+bx^2\right)^7$ | |
| | | | | **[4]** | | |
| 5 | | | Attempt to connect d$u$ and d$x$ or find $\frac{du}{dx}$ | M1 | no accuracy ; not $du=dx$ | |
| | | | $du=-\sin x\ dx$ or $\frac{du}{dx}=-\sin x$ | A1 | | |
| | | | Indefinite integral becomes $-\int\left(1-u^2\right)u^2\ (du)$ | A1 FT | FT only from $\frac{du}{dx}=\sin x$ | |
| | | | $-\int\left(1-u^2\right)u^2\ \ (du)=-\frac{1}{3}u^3+\frac{1}{5}u^5$ | B1 | Award also for $\int\left(1-u^2\right)u^2\ du=\frac{1}{3}u^3-\frac{1}{5}u^5$ | |
| | | | Use new limits if f($u$) or original limits if resubstitution | M1 | no accuracy | |
| | | | $\frac{47}{480}$ AE Fraction | A1 | ISW **www** If A0, answer of 0.0979… → M1 | |
| | | | | **[6]** | | |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| **6** | | | State or imply that graphs cross at $x = \frac{1}{4}\pi$ | B1 | (Limits on integrals may clarify) | Be lenient here |
| | | | $\pi\int y^2 \mathrm{d}x$ used with either $y = \sin x$ or $y = \cos x$ | *M1 | The ' $\pi$ ' element(s) may not appear until later in the working. | |
| | | | $\pi\int_0^{\frac{1}{4}\pi} \sin^2 x \ (\mathrm{d}x) + \pi\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} \cos^2 x \ (\mathrm{d}x)$ or $2\pi\int_0^{\frac{1}{4}\pi} \sin^2 x \ (\mathrm{d}x)$ | A1 | | |
| | | | Changing $\sin^2 x$ or $\cos^2 x$ into f(cos 2$x$) | **dep*M1** | | |
| | | | $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$ or $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ | A1 | | |
| | | | $\int \cos 2x \ (\mathrm{d}x) = \frac{1}{2} \sin 2x$ anywhere in this part | B1 | | |
| | | | $\frac{1}{4}\pi^2 - \frac{1}{2}\pi$ | A1<br>**[7]** | ISW | |
| **7** | (i) | | Use $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1+t \\ -t \\ 2 \end{pmatrix}$ | B1 | | |
| | | | $\begin{pmatrix} x \\ y \\ z \end{pmatrix} . \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix} = 0$ | M1 | | |
| | | | $\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 2 \end{pmatrix}$ or $\frac{1}{2}\mathbf{i} + \frac{1}{2}\mathbf{j} + 2\mathbf{k}$ | A1<br>**[3]** | | |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| 7 | (ii) | | $(1+t)^2+t^2+4=3^2$ or $\sqrt{(1+t)^2+t^2+4}=3$ | M1 | FT from their **(i)** $P$ | |
| | | | $t=1$ or $-2$ | A1 | SR If A0A0 award A1A0 for either value of $t$ leading to its correct answer. | |
| | | | $\begin{pmatrix}2\\-1\\2\end{pmatrix}$ and $\begin{pmatrix}-1\\2\\2\end{pmatrix}$ | A1 | | |
| | | | | **[3]** | | |
| 8 | (i) | | $\frac{dy}{dx}=\frac{\text{attempt at }\frac{dy}{d\theta}}{\text{attempt at }\frac{dx}{d\theta}}$ but not $\frac{4-3\sin^2\theta}{2\sin\theta}$ | M1 | | Alternative<br>Change to Cartesian form, differentiate and resubstitute<br>Correct differentiation of correct equation |
| | | | $4\cos\theta-3\sin^2\theta\cos\theta$ seen | B1 | indep | |
| | | | $\left(\frac{dy}{dx}=\right)\frac{4\cos\theta-3\sin^2\theta\ \cos\theta}{2\sin\theta\cos\theta}=\frac{4-3\sin^2\theta}{2\sin\theta}$ **AG** | A1 | | |
| | | | | **[3]** | | |
| 8 | (ii) | | Equating given $\frac{dy}{dx}$ to 2 & producing quadratic equation | M1 | | |
| | | | $\sin\theta=\frac{2}{3}$ | A1 | ignore any other given value | |
| | | | $P$ is $\left(\frac{4}{9},\frac{64}{27}\right)$ | A1 | Accept 0.444… and 2.37… or better | |
| | | | | **[3]** | | |
| 8 | (iii) | | Identify problem as solving $4-3\sin^2\theta=0$ | M1 | | |
| | | | Show convincingly that $4-3\sin^2\theta=0$ has no solutions | A1 | Consider magnitude of $\sin\theta$ | |
| | | | | **[2]** | | |
| 8 | (iv) | | Attempt to eliminate $\sin\theta$ from the 2 given equations | M1 | e.g. $y=4\sqrt{x}-\left(\sqrt{x}\right)^3$ | |
| | | | Produce $y^2=x(4-x)^2$ or $16x-8x^2+x^3$ | A1 | ISW | |
| | | | | **[2]** | | |

| Question | | | Answer | Marks | Guidance | |
|---|---|---|---|---|---|---|
| 9 | | | Use $u = x^2 + 1,\ \mathrm{d}v = \mathrm{e}^{2x}$ or $u = x^2,\ \mathrm{d}v = \mathrm{e}^{2x}$ | M1 | 1st stage = f($x$)+/- $\int \mathrm{g}(x)\ \mathrm{d}x$ | |
| | | | 1st stage= $\frac{1}{2}(x^2+1)\ \mathrm{e}^{2x} - \int x\ \mathrm{e}^{2x}\ \mathrm{d}x$ or $\frac{1}{2}x^2\mathrm{e}^{2x} - \int x\mathrm{e}^{2x}\mathrm{d}x$ | A1 | | |
| | | | For $\int x\ \mathrm{e}^{2x}\ \mathrm{d}x$, use $u = x,\ \mathrm{d}v = \mathrm{e}^{2x}$ | M1 | ditto | |
| | | | $= \frac{1}{2}x\ \mathrm{e}^{2x} - \frac{1}{4}\ \mathrm{e}^{2x}$ | A1 | tolerate second sign error in $-\int x\mathrm{e}^{2x}\mathrm{d}x$ | |
| | | | Complete final stage = $\frac{1}{2}(x^2+1)\ \mathrm{e}^{2x} - \frac{1}{4}(2x-1)\ \mathrm{e}^{2x}$ | A1 | soi; may be separate terms | |
| | | | Correct (method) use of limits seen anywhere | M1 | Do not accept (…..) − 0 | |
| | | | Final answer = $\frac{3}{4}\mathrm{e}^2 - \frac{3}{4}$ | A1 | ISW; if A0, answer of 4.79… → M1 | |
| | | | | **[7]** | | |
| 10 | (i) | | $\frac{1}{2}(y^2+1)^{-\frac{1}{2}}.2y$ or better | B1 **[1]** | Tolerate "$\frac{\mathrm{d}y}{\mathrm{d}x}$ = …" but, otherwise, no $\frac{\mathrm{d}y}{\mathrm{d}x}$ or $\frac{\mathrm{d}x}{\mathrm{d}y}$ | |
| 10 | (ii) | | Separate variables; $\int \frac{y}{\sqrt{y^2+1}}\ \mathrm{d}y = \int \frac{x-1}{x}\ \mathrm{d}x$ | *M1 | $\int$ may be implied later | |
| | | | Change $\frac{x-1}{x}$ into $1 - \frac{1}{x}$ | M1 | | |
| | | | RHS = $x - \ln x$ | A1 | | |
| | | | LHS = $\sqrt{y^2+1}$ | B1 | Quoted or derived | |
| | | | Subst $y = \sqrt{\mathrm{e}^2 - 2\mathrm{e}},\ x = \mathrm{e}$ into their eqn. with '$c$' | Dep*M1 | | |
| | | | $\sqrt{y^2+1} = \sqrt{(\mathrm{e}-1)^2} = \mathrm{e} - 1$ | A1 | Ignore lack of/no ref to $1-\mathrm{e}$ | |
| | | | $c = 0$ | A1 | Ignore any ref to $c = 2 - 2\mathrm{e}$ | |
| | | | $\sqrt{y^2+1} = x - \ln x$ | A1 | ISW | |
| | | | | **[8]** | | |